# Scanning Our Past

## Origins of the Equivalent Circuit Concept: The Current-Source Equivalent

### I. Introduction

As described in my previous paper [1], the voltage-source equivalent was first derived by Hermann von Helmholtz (1821–1894) in an 1853 paper [2]. Exactly thirty years later in 1883, Léon Charles Thévenin (1857–1926) published the same result [3], [4] apparently unaware of Helmholtz's work. The generality of the equivalent source network was not appreciated until forty-three years later. Then, in 1926, Edward Lawry Norton (1898–1983) wrote an internal Bell Laboratory technical report [5] that described in passing the usefulness in some applications of using the current-source form of the equivalent circuit. *In that same year*, Hans Ferdinand Mayer (1895–1980) published the same result [6] and detailed it fully. As detailed subsequently, these people intertwine in interesting ways.

### II. Mayer

Hans Ferdinand Mayer was born on October 23, 1895, in Pforzheim, Germany, which is located halfway between Stuttgart and Karlsruhle. After receiving a leg wound in his first action in World War I (1914), he studied physics and mathematics at the Technische Hochschule in Stuttgart and went on to the University of Heidelberg to become a research assistant to Philipp Lenard (1862–1947), a Nobel Prize winner in physics (1905). He received his doctorate in 1920, with his dissertation concerning the interaction of slow electrons with molecules. He continued working as a research assistant for Lenard until 1922, and then joined Hause-Siemens. He became Director of Siemens Research Laboratory in 1936. Except for interludes during and after World War II, he worked for Siemens until his retirement in 1962. He published 25 technical papers during his life and secured over 80 patents. He received an honorary doctorate from the Technische Hochschule in Stuttgart in 1956, the Gauss–Weber Medal from the University of Göttingen, the Philipp Reis award from the German Post Office in 1961, and the Ring of Honor from the VDE in 1968 [7]. Mayer died on October 16, 1980, in Munich.

Short biographies of Mayer are provided in [7]–[9], with [7] listing his publications. As recognized as he was for his technical work, Mayer's personal life perhaps had more impact. As described in [8], [10], [11], Mayer secretly leaked to the British in November 1939 all he knew of Germany's warfare capabilities, particularly concerning electronic warfare. Because he represented Siemens as a technical expert in negotiations with companies outside Germany, he had the opportunity to travel widely about Europe. While in Oslo, Norway, he typed and mailed a two-page report of what he knew and mailed it to the British Embassy in Oslo. Because Mayer wrote it anonymously, the British, led by Reginald Jones, had to determine the report's accuracy. Jones found what became known as the Oslo Report to be a technically knowledgeable person's description of what he/she knew (although it contains some errors) [11]. Only after the war did Jones determine that Mayer was the "Oslo Person." Mayer did not even tell his family of his role in the Oslo Report until 1977 [10]. He requested that his contribution not be made known until after his and his wife's death. Jones described Mayer's contributions in 1989 [8] and a newspaper feature appeared that same year [10]. During the war, Mayer continued working at Siemens, until he was arrested in 1943 by the Gestapo for listening to the BBC and speaking out against the Nazi regime.[1] He was saved from execution by his doctoral advisor Lenard, despite Lenard being a strong supporter of the Nazis (he first met Hitler in 1926) and being anti-Semitic to the extreme (so much so he could not believe any Jew's physics, Einstein in particular). Mayer was put into the Dachau concentration camp, and later moved into four others during the remaining years of the war. After the war, he joined the electronics research effort at Wright-Patterson Air Force Base, Dayton, OH, which at the time was the U.S. Air Force's primary research laboratory. He left the laboratory in 1947 to become Professor of Electrical Engineering at Cornell University [12]. It is during this time he wrote his letter describing Helmholtz's role in developing equivalent circuits [13]. After the Federal Republic was established in 1949 and Siemens was returning to its pre-war prominence, he returned to Germany in 1950 to work with Siemens in Munich.

In November 1926, Mayer published a paper [6] that describes the conversion of the voltage-source equivalent circuit to a parallel combination of a current source and the equivalent impedance (Fig. 2). This paper makes no refer-

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[1]The Nazis were never aware of the Oslo Report.

ence to Helmholtz or Thévenin; in fact, he refers to both forms as "Ersatzschema" (equivalent circuits). He derives the current-source equivalent circuit by simply noting that it has the same terminal behavior as the voltage-source equivalent (similar to the proof given in my previous paper [1]). His concern was finding the equivalent circuit for the output of electronic amplifiers. Mayer is perhaps the first to point out that the equivalent voltage and current source values equal the open-circuit voltage and short-circuit current respectively. His paper is about two and one-half pages long, with about a page of it an editor's comment. His portion is divided into five numbered sections, two of which are translated here.

1) [A review of vacuum tube amplifiers and how the voltage-source equivalent reflects their characteristics.]
2) Consider first the simple case in Fig. 1 [referring to Abb. 1 in this paper's Fig. 2], where an electromotive source $E$ with an internal resistance $R_i$ is connected to an external resistance $R_a$. Such an arrangement is perfectly equivalent from the viewpoint of $R_a$ to that shown in Fig. 2 [Abb. 2 in Fig. 2], where the electromotive source is replaced by a current source $J = E/R_i$, because in both cases the voltage $V$ results. From the viewpoint of $R_a$, the circuit it is attached to can be characterized two different ways: either, as in Fig. 1 [Abb. 1], as a electromotive source $E$ and an internal resistance $R_i$ or, as in Fig. 2 [Abb. 2], as a current source $J$ and an internal conductance $G_i = 1/R_i$. As with the electromotive source $E$ in Fig. 1 [Abb. 1], the current source $J$ in Fig. 2 [Abb. 2] does not depend on outside resistances [loads]. The source value $E$ is identical to the open-circuit voltage $V$ and $J$ equals the short-circuit current.
3) [A reinterpretation of Section II in terms of vacuum tube amplifiers.]
4) [An example of a parallel loading circuit showing that the current source equivalent makes calculations simpler.]
5) The reciprocity between the networks shown in Figs. 1 and 2 [Abb. 1 and 2] can be extended to any network. Assume a network has resistances $R_1, \ldots, R_n$ and equivalent electromotive sources $E_1, \ldots, E_n$. If one wants to calculate the current flowing through any resistance, then the following way proves fruitful: First imagine a particular resistance is isolated and calculate the open-voltage occurring in its place. The remaining network can be replaced by one source, its electromotive force equal to $E$, and its internal resistance is the resistance of the network seen from the particular resistance's viewpoint if all the electromotive forces $E_1, \ldots, E_n = 0$.

   The other equivalent source of Fig. 2 [Abb. 2] would be found by replacing the particular resistance by a short circuit and measuring the current $J$ that flows. The network from the particular resistance's viewpoint can be replaced by a single current source having a value equal to the short-circuit current $J$. The internal conductance $G$ of this equivalent network is that seen from the particular resistance if all the electromotive sources to zero. The relationship between the resistance $R$ and the conductance $G$ is $R \cdot G = 1$.

   [The paper concludes with a long remark by the editor Hermann Schulz showing that the equivalent extends to complex amplitudes sources and impedances. He also shows how the Norton equivalent can be usefully applied in a two-port example.]

Mayer's description is quite clear and contains all the central ideas found in modern presentations of equivalent circuits. Wallot's 1932 textbook describes the current-source equivalent and references Mayer [14]. To my knowledge, no textbook written by American authors mentions Mayer.

## III. NORTON

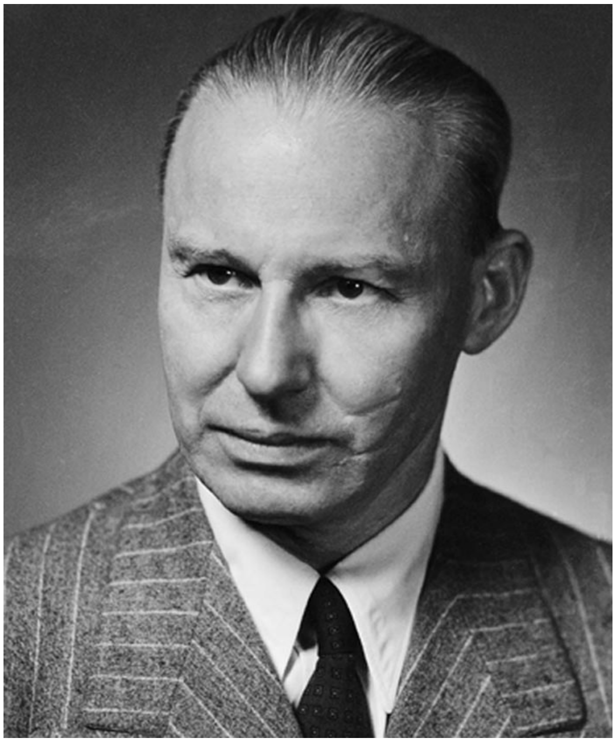
No biography was ever written about Norton; what follows was obtained from the AT&T Archives. Detailed information about Norton can be found at the author's web site (http://www.ece.rice.edu/~dhj/Norton).

Edward Lawry Norton was born on July 29, 1898, in Rockland, ME. He served as a radio operator in the U.S. Navy between 1917 and 1919. He attended the University of Maine for one year before and for one year after his wartime service, then transferred to M.I.T. in 1920, receiving his S.B. degree (electrical engineering) in 1922. He then joined the Western Electric Company (the predecessor to Bell Telephone Laboratories) and received his masters' degree in electrical engineering from Columbia University in 1925. He remained with Bell Labs all of his career, retiring in 1963. He died on January 28, 1983, at the King James Nursing Home in Chatham, NJ.

During his forty-one year career at Bell Labs, he wrote only three technical papers [15]–[17], none of which concerned or mentioned the equivalent circuit that bears his name today. During his career he obtained 18 patents,[2] which also contain no mention of his equivalent circuit. He wrote 92 technical reports during his career, and in one of these, *Design of Finite Networks for Uniform Frequency Characteristic*, dated November 3, 1926, a short paragraph describing the current-source equivalent appears [5].

> The illustrative example considered above gives the solution for the ratio of the input to output current, since this seems to be of more practical interest. An electric network usually requires the solution for the case of a constant voltage in series with an output impedance connected to the input of the network. This condition

[2] A typed biography form in the AT&T archives dated July 20, 1954, states that he had 19 patents. A handwritten biography form dated two years later states "approximately twenty." Only 18 could be found in the U.S. Patent and Trademark Office records.



(a)

(b)

Fig. 1. (a) Mayer's photograph was taken about 1940 and comes from the historical photograph collection provided on a CD by the SiemensForum (http://www.siemens.de/siemensforum). (b) Norton's photograph has been provided by the AT&T Archives and is dated October 13, 1925.

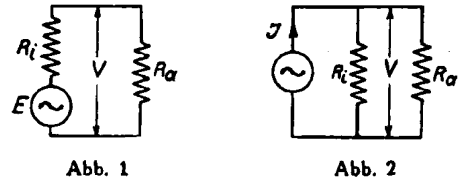



Fig. 2. Reproduction of the critical figure from Mayer's paper.

> would require the equations of the voltage divided by the current in the load to be treated as above. It is ordinarily easier, however, to make use of a simple theorem which can be easily proved, that the effect of a constant voltage $E$ in series with an impedance $Z$ and the network is the same as a current $I = E/Z$ into a parallel combination of the network and the impedance $Z$. If, as is usually the case, $Z$ is a pure resistance, the solution of this case reduces to the case treated for the ratio of the two current, with the additional complication of a resistance shunted across the input terminals of the network. If $Z$ is not a resistance the method still applies, but here the variation of the input current $E/Z$ must be taken into account.

During the time of this technical report, Norton was working on circuit design and on electrical models for phonographs [18]; he is mentioned in the paper [19, Footnote 7] that resulted from this work. Note that Norton's technical report is dated the same *month* as Mayer's publication.

Because of Norton's lack of publications, it appears that Norton preferred working behind the scenes. As described in the history of Bell Labs [20, p. 210], this reticence belied his capabilities.

> Norton was something of a legendary figure in network theory work who turned out a prodigious number of designs armed only with a slide rule and his intuition. Many anecdotes survive. On one occasion T.C. Fry called in his network theory group, which included at that time Bode, Darlington and R.L. Dietzold among others, and told them:
>
> "You fellows had better not sign up for any graduate courses or other outside work this coming year because you are going to take over the network design that Ed Norton has been doing single-handed."

He applied his deep knowledge of circuit analysis to many fields, and after World War II he worked on Nike missile guidance systems [21]. Norton became a Fellow of the Acoustical Society of America and a Fellow of the Institute of Radio Engineers in 1961.

How Norton's name became associated with the equivalent circuit that bears his name is murky. It is not mentioned (but Thévenin is) in the 1929 classic book [22, pp. 55–56] by Shea, who worked at Bell Laboratories. The Timbie and Bush textbook edition from 1940 mentions Thévenin but not Norton or his equivalent; their 1951 edition does [23]. Smith's 1949 textbook clearly describes the current-source equivalent but without reference [24]. The 1940 publication of the book derived from teaching the first course in circuit analysis at MIT mentions the voltage–source equivalent but not the current-source equivalent [25]; however, on p.

145 a tantalizing footnote describes both Thévenin's and Helmholtz's contributions and ends with

> The theorem is actually more general than stated for the special case of resistance [sic] networks. It can be stated in terms of current source and conductance if desired.

## IV. Conclusion

As frequently occurs in science and engineering, the name associated with a law or concept may not have been the first or even primary responsible person. Clearly, more than two people deserve credit for developing the equivalent circuit concept. Helmholtz clearly originated the voltage-source equivalent. I can only presume that his paper's title (concentrating on animal electricity) and its publication early in his career meant it was not read by electrical scientists despite Helmholtz's eventual scientific stature. Helmholtz's biography [26], [27] describes the critical portion of his paper, but makes no allusion to Thévenin or its engineering importance. That said, in Europe, Helmholtz's name is associated with the equivalent circuit (any Web search will reveal this fact). Mayer developed the current-source version more fully and more publicly than did Norton. However, Mayer's work was published in a somewhat obscure German technical journal. I have not been able to determine just how Norton's name became associated with the current-source equivalent. Again, in Europe, both people are associated with it, where the equivalents are known by various combinations of these four person's names: Helmholtz–Thévenin, Helmholtz–Norton, Mayer–Norton, etc.

The current-source equivalent did not occur to early electrical scientists because of the seeming impossibility of a current source existing. An ideal current source will produce a specified current no matter what is attached to it, be the attached element an open circuit (which means it produces a controlled current into free space) or a short circuit (current flows through an ideal wire without dissipating any more heat than when attached to a nonzero impedance). Only later did Norton and Mayer realize that the current-source equivalent was easier to use in theoretical work in certain situations (e.g., when the load consists of a parallel combination). We now recognize that it also provides more insight into the circuit when the equivalent impedance is larger than the load, in which case the current flow is approximately constant across variations in load impedance.

Despite the current-source equivalent being taught to all electrical engineers in their first course, Norton labored in relative obscurity. He was very well known by those at Bell Laboratories. Telephone calls to now-retired Bell Laboratories researchers I know revealed that all recalled him and confirmed his stature at the Lab. Presumably one or several of Norton's colleagues credited the current-source equivalent to him some time before 1950.

Hans Ferdinand Mayer deserves more recognition in the United States than he has now. His contribution was published and, because journal publication delays usually exceed those of a technical report, presumably discovered the utility of the current-source equivalent earlier than Norton. His journal, *Telegraphen- und Fernsprech-Technik*, was then and now not well known; this fact certainly contributed to an unawareness of his work. His wartime actions were courageous but not well-known in the United States. Perhaps future textbooks should follow Mayer's suggestion described in [1]: Credit the voltage-source equivalent to Thévenin. I would suggest that the current-source equivalent be named the Mayer–Norton equivalent.

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